THE NEUBER’S RULE AND A CORRELATION BETWEEN FRETTING FATIGUE AND PLAIN FATIGUE

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ABSTRACT

Based on these experimental observations and on the remark that any small change on the discontinuity geometry has a significant effect on the stress distribution around it, a stress concentration factor is mandatory to be incorporated into the fatigue models in order to accurately predict the life. In this paper, it is proposed a correlation between the global degradation processes in plain fatigue case and the local degradation process in fretting fatigue case.

Keywords: stress concentration factor, fatigue, fretting fatigue, Neuber’s rule, SWT model

1. INTRODUCTION

It is well known that fatigue, which is the most frequent mechanics that deal with failure analysis, arises due to repeated loading and involves the gradual development and growth of a crack [1, 2]. When the synergetic effect (fatigue plus fretting) exists, there occurs one of the most damaging phenomena, known as fretting fatigue. Fretting is a special process that occurs in the contact area between two components under load giving rise to a slight relative displacement due to vibration or some other forces [3].

In a practical application, by designing the appropriate geometry (plain fatigue)/contact geometry (fretting fatigue), stress concentration and consequently the fatigue cracks can be avoided. For example, in a study [4] about an automotive-formed suspension component it was shown that a significant improvement in fatigue life could be achieved with a small change on the contact geometry (introducing a curvature radius on the head of the original screw). This small change on the geometry reduced locally the stress level and increased the relative displacement amplitude and consequently fatigue life.

It is well known that in fatigue case any small changes on geometrical (for example: holes, notches, fillets and threads) or microstructural discontinuities have a significant effect on the stress distribution around it. The stresses around the changes on geometrical (for example: holes, notches, fillets and threads) often exceed the nominal elastic stresses, and under fatigue loading cyclic inelastic strains in the area of stress and strain concentrations may cause formation of cracks and this could lead to significant reduction on life.

In the case of fretting fatigue the situation is more complicated due to fact that besides the nominal axial (machine) stress (the fatigue) there are also the normal and tangential contact stresses. In a previous study on an aluminium alloy [5], it has been shown that when the fretting fatigue tests were concluded, a concavity was formed at the fretted surface of the specimen due to the contact of the fretting pads. Also, it was observed the variation of the depth of the fretting scar with normal or tangential load. The depth increased linearly with the increasing of the normal load and with tangential load. Based on these experimental observations and on the remark that any small change on the discontinuity geometry has a significant effect on the stress distribution around it, a stress concentration factor is determined as a function of the fretting scar depth. Thus, a new parameter was introduced into the SWT model, which is a stress concentration factor, $K_f$, that incorporates the effect of the damaged area, called the “fretting scar effect”. SWT model was chosen because it is a very popular model in fatigue case [6-8] and also in the fretting fatigue case [9-13], considered among the most reliable one, and also this model that includes a good quantity of intrinsic material properties being then more appropriate to understand either the influence of material property on fatigue life as well as to establish comparisons among different materials. The results of the same study indicated that the SWT parameter (without $K_f$) gives a good correlation between predicted life and experimental results. However, a
better correlation was obtained by introducing the stress concentration factor on the SWT parameter.

In the present paper it is proposed a correlation between the global degradation processes in plain fatigue case and the local degradation process in fretting fatigue case.

2. THEORETICAL BACKGROUND

2.1. Smith Watson Topper (SWT) model

a. Initial remarks

Figure 1 shows schematically the strain-life fatigue curves. The total fatigue strain amplitude in Figure 1 has been resolved into elastic and plastic strain components. As it is already known at a given life, the total fatigue strain amplitude is the sum of the elastic and plastic strains. The empirical relationship used to define the total number of cycles to fatigue failure of metallic materials can be expressed in the following form:

\[ \varepsilon_a = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\sigma_f'}{E} (2N_f)^b + \varepsilon_f' (2N_f)^c \]  

(1)

where \( \varepsilon_a \) - total fatigue strain amplitude; \( \frac{\Delta \varepsilon_e}{2} \) - elastic strain amplitude; \( \frac{\Delta \varepsilon_p}{2} \) - plastic strain amplitude; \( \sigma_f' \) - is the fatigue strength coefficient; \( E \) is the Young's modulus; \( N_f \) is the number of cycles to crack initiation; \( b \) is fatigue strength exponent; \( \varepsilon_f' \) is the fatigue ductility coefficient; \( c \) is the fatigue ductility exponent.

Dividing the Basquin's equation [15] by the modulus of elasticity gives the equation for the elastic strain amplitude-life curve:

\[ \frac{\Delta \varepsilon_e}{2} = \frac{\Delta \sigma}{2E} = \frac{\sigma_a}{E} = \frac{\sigma_f'}{E} (2N_f)^b \]  

(2)

The relation between plastic strain and life (the Coffin-Manson’s equation) is represented by

\[ \frac{\Delta \varepsilon_p}{2} = \varepsilon_f' (2N_f)^c \]  

(3)

Equation (1) does not consider eventual mean stresses during cyclic loading [15]. It should be mentioned that the predicted analytical life is the initiation life.

b. SWT in the plain fatigue case and Neuber's rule

Smith et al. [16] proposed a suitable relation that includes both the cyclic strain range and the maximum stress. This model, commonly referred to as the SWT parameter, was originally developed and continues to be used as a correction for the machine load mean stresses in uniaxial loading conditions [16].

The SWT parameter for multiaxial loading is based on the total fatigue strain amplitude, \( \varepsilon_a \) and the maximum tensile stress, \( \sigma_{max} \) normal to the crack plane during a loading cycle it is expressed in the following mathematical form

\[ SWT = \sigma_{max} \cdot \varepsilon_a \]

The maximum tensile stress value is for plane alternating fatigue test and it is given by

\[ \sigma_{max} = \frac{\Delta \sigma}{2} = \sigma_f'(2N_f)^b \]  

(5)

and by multiplying the strain-life equation (1), the SWT mean stress correction formula is expressed as follows:

\[ \sigma_{max} \cdot \varepsilon_a = \frac{\sigma_f'}{E} (2N_f)^{2b} + \sigma_f' \varepsilon_f' (2N_f)^{b+c} \]  

(6)

Neuber [1, 3, 7, 17-20] hypothesized that the elastic stress concentration factor \( (K_e) \) is the geometric means of the true stress and strain concentration factors:

\[ K_e = \sqrt{K_{\sigma}K_{e}} \]  

(7)

This approach works very well for situations in which only the elastic stresses and strains are presented. However, most components may appear to have nominally cyclic elastic stresses, but some stress concentrations present in component result in local cyclic plastic deformation. Under these conditions, another approach, proposed by Topper [15] recommended to use the fatigue notch factor, \( K_f \) instead of the theoretical stress concentration factor.

The fatigue notch factor, \( K_f \) can be determined with the following relation [21,22]:

\[ K_f = 1 + q \cdot (K_e - 1) \]  

(8)

where \( q \) – fatigue notch sensitivity

The fatigue notch sensitivity \( q \) is the measure of the degree of agreement between \( K_f \) and \( K_e \).

According to Neuber

\[ q = 1 + \frac{\rho}{r} \]  

(9)

Thus,

\[ K_f = 1 - \frac{K_e - 1}{\sqrt[1+\rho]{r}} \]  

(10)

where \( \rho \) is a material characteristic length, \( r \) is the notch root radius.

Then Neuber’s rule (7) can be expressed as
where $\sigma_{\text{max}}$ - maximum stress; $\sigma_n$ - the nominal stress; $\varepsilon$ - maximum strain; $\varepsilon_n$ - the nominal strain.

By substituting the nominal strain, $\varepsilon$, eq. (11) can be written as

$$K_f = \frac{\sigma_{\text{max}} \cdot \varepsilon}{\sigma_n \cdot \varepsilon} \varepsilon_n$$  \hspace{1cm} (12)

Equation (12) can be rewritten as follows:

$$K_f \cdot \sigma_n = \sqrt{\frac{\sigma_{\text{max}} \cdot \varepsilon}{\varepsilon_n}}$$  \hspace{1cm} (13)

If in relation (13) the right side is substituted by the corresponding value from equation (6)

$$K_f \cdot \sigma_n = \left[\left(\frac{\sigma_f}{\varepsilon}\right)^2 (2N_f)^{2b} + \sigma_f \varepsilon_f (2N_f)^{b+c}\right]^{1/2}$$  \hspace{1cm} (14)

c. SWT for the fretting fatigue case and Neuber's rule

Prior to the application of the SWT parameter to the fretting fatigue case, some corrections were introduced in order to take into consideration all the stresses involved in the contact. As it is already known, the crack initiation occurs on the plane (critical plane) where the product between the maximum stress and the total strain amplitude is maximum. So, the maximum fretting fatigue stress ($\sigma_{\text{max,FF}}$) normal to the trailing edge of the contact can be estimated as a superposition of the fretting contact stresses and the machine applied axial stress.

The SWT parameter that can be applied to the fretting fatigue situation takes the following form:

$$\sigma_{\text{max,FF}} \cdot \varepsilon_{a,FF} = \left(\frac{\sigma_f}{\varepsilon}\right)^2 (2N_f)^{2b} + \sigma_f \varepsilon_f (2N_f)^{b+c}$$  \hspace{1cm} (15)

where:

$$\sigma_{\text{max,FF}} = \left(\sigma_{\text{max}} + 2p_0\sqrt{\mu F_{t,\text{max}}/F_{n,\text{max}}} \right)$$  \textit{is the maximum fretting fatigue stress,}

$$\varepsilon_{a,FF} = \frac{1-2\mu^2}{E} \left(\sigma_a + \sigma_{a,F_f} \right)$$  \textit{is the total fretting fatigue strain amplitude; $\sigma_{\text{max}}$ is the maximum machine axial stress, $p_0$ is the maximum Hertzian pressure, $\mu$ is the coefficient of friction in the slip condition, $F_{n,\text{max}}$ is the maximum normal contact load, $F_{t,\text{max}}$ is the maximum tangential load, $\nu$ is the Poisson’s ratio, $\sigma_a$ is the machine axial stress amplitude; $\sigma_{a,F_f}$ is the tangential stress amplitude.}

A more detailed theoretical background on the SWT model is presented elsewhere [5].

In fretting fatigue case, the stress increases because of the fretting scar that is formed and this affects the stress distribution near it. Therefore, the maximum stress alone no longer describes the global state of stress in the part as it has been shown in a previous work [14], about the material response at the fretting scar (damaged area). Thus, a new parameter was introduced into the SWT model, which is a stress concentration factor, $K_f$.

Thus, equation (15) becomes

$$\sigma_{\text{max,FF}} \cdot \varepsilon_{a,FF} \cdot K_f = \frac{\sigma_f^2}{E} (2N_f)^{2b} + \sigma_f \varepsilon_f (2N_f)^{b+c}$$  \hspace{1cm} (16)

3. RESULTS AND DISCUSSIONS

3.1. Stress Concentration Factor Effect on Predicting Fretting Fatigue Life by SWT Parameter

Figure 2 shows the analytical fretting fatigue life prediction based on the SWT parameter which takes into consideration only the maximum stress effects (eq. 15 - dashed red line). It also shows the estimated life obtained using the SWT parameter affected by the stress concentration factor, $K_f$ (Neuber), (eq. 16 – solid green line).

The main aspect to highlight is that when the stress concentration factor is incorporated into the SWT parameter, good predictions are obtained, in the case of Al7175 alloy.

Regarding the use of the stress concentration factor on the proposed modified SWT parameter, it is observed that it may have a substantial effect on fretting fatigue predictions. As a fact, it is well known that the stress concentration factor is very important in the case of fatigue loading. If the specimen is subjected to an axial tensile or compressive load, the stress is assumed to be the same across the section. However, in the presence of any sudden change of section (hole, sharp corner, notch, etc.), the local stress will rise significantly [15]. The stress increases because of the fretting scar that is formed which affects the stress distribution near it. Therefore, the maximum stress alone no longer describes the global state of stress in the part.
3.2. Proposed Approach

In order to establish a relationship between the global degradation processes by plain fatigue (equation 14) and the local degradation processes by fretting fatigue (eq. 16), the following simplifications are considered:

- the term \(\frac{(\sigma_f)^2}{E}(2N_f)^{2b} + \sigma_f\varepsilon_f(2N_f)^{b+c}\) from the right-hand side of equations (14) and (16) is replaced by \(X\):

\[
\frac{(\sigma_f)^2}{E}(2N_f)^{2b} + \sigma_f\varepsilon_f(2N_f)^{b+c} = X
\]  

(17)

- and, the product between the maximum stress and the total strain amplitude from the left-hand of equation (16) is replaced by

\[
\sigma_{\text{max}} + 2\rho_\text{av}\left(\frac{\mu F_{t,max}}{F_{n,max}}\right)^{\frac{1}{2}}(\sigma_a + \sigma_{a,K}) = \sigma_{\text{fret}}
\]  

(18)

It can be seen that equation (19) is comparable with equation (20).

For the same number of cycles and the same material, the \(\frac{X}{\sigma_f}\) value is the same in both cases: plain fatigue and fretting fatigue. In this situation, it can be supposed that the fatigue stress is equivalent to the fretting fatigue stress and it can be obtained from the following equations:

- fatigue case

\[K_f \cdot \sigma_n = \left(\frac{X}{\sigma_f}\right)^{1/2}
\]  

(19)

- fretting fatigue case

\[K_f \cdot \sigma_{\text{fret}} = X
\]  

(20)

In this equations, \(\sigma_n\) can be defined as a fatigue stress equivalent to fretting fatigue and it will be named: \(\sigma_{eq}\).

Thus, the equation proposed to establish a correlation between the global degradation processes in plain fatigue case and the local degradation process in fretting fatigue case can be written as follows:

\[\sigma_{\text{fret}} = K_f^2 \cdot \frac{\sigma_n^2}{K_n} \cdot \sigma_{eq}
\]  

(21)

4. CONCLUSIONS

The conclusions drawn from this work are as follows:

- In the case of cyclic loading the localized plastic deformation is the main cause of the micro-degradations, which leads in most of the cases to catastrophic failure of the component/specimen. These plastic deformations can be determined by using the stress concentration factor.

- SWT model and Neuber’s rule allow to establish a correlation between the global degradation processes in plain fatigue case and the local degradation process in fretting fatigue case.

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REFERENCES